

## CHAPTER 1: Introduction, Measurement, Estimating

### Answers to Questions

1. (a) Fundamental standards should be accessible, invariable, indestructible, and reproducible. A particular person's foot would not be very accessible, since the person could not be at more than one place at a time. The standard would be somewhat invariable if the person were an adult, but even then, due to swelling or injury, the length of the standard foot could change. The standard would not be indestructible – the foot would not last forever. The standard could be reproducible – tracings or plaster casts could be made as secondary standards.  
(b) If any person's foot were to be used as a standard, "standard" would vary significantly depending on the person whose foot happened to be used most recently for a measurement. The standard would be very accessible, because wherever a measurement was needed, it would be very easy to find someone with feet. The standard would be extremely variable – perhaps by a factor of 2. That also renders the standard as not reproducible, because there could be many reproductions that were quite different from each other. The standard would be almost indestructible in that there is essentially a limitless supply of feet to be used.
2. There are various ways to alter the signs. The number of meters could be expressed in one significant figure, as "900 m (3000 ft)". Or, the number of feet could be expressed with the same precision as the number of meters, as "914 m (2999 ft)". The signs could also be moved to different locations, where the number of meters was more exact. For example, if a sign was placed where the elevation was really 1000 m to the nearest meter, then the sign could read "1000 m (3280 ft)".
3. Including more digits in an answer does not necessarily increase its accuracy. The accuracy of an answer is determined by the accuracy of the physical measurement on which the answer is based. If you draw a circle, measure its diameter to be 168 mm and its circumference to be 527 mm, their quotient, representing  $\pi$ , is 3.136904762. The last seven digits are meaningless – they imply a greater accuracy than is possible with the measurements.
4. The problem is that the precision of the two measurements are quite different. It would be more appropriate to give the metric distance as 11 km, so that the numbers are given to about the same precision (nearest mile or nearest km).
5. A measurement must be measured against a scale, and the units provide that scale. Units must be specified or the answer is meaningless – the answer could mean a variety of quantities, and could be interpreted in a variety of ways. Some units are understood, such as when you ask someone how old they are. You assume their answer is in years. But if you ask someone how long it will be until they are done with their task, and they answer "five", does that mean five minutes or five hours or five days? If you are in an international airport, and you ask the price of some object, what does the answer "ten" mean? Ten dollars, or ten pounds, or ten marks, or ten euros?
6. If the jar is rectangular, for example, you could count the number of marbles along each dimension, and then multiply those three numbers together for an estimate of the total number of marbles. If the jar is cylindrical, you could count the marbles in one cross section, and then multiply by the number of layers of marbles. Another approach would be to estimate the volume of one marble. If we assume that the marbles are stacked such that their centers are all on vertical and horizontal lines, then each marble would require a cube of edge  $2R$ , or a volume of  $8R^3$ , where  $R$  is the radius of a marble. The number of marbles would then be the volume of the container divided by  $8R^3$ .

7. The result should be written as 8.32 cm. The factor of 2 used to convert radius to diameter is exact – it has no uncertainty, and so does not change the number of significant figures.
8.  $\sin 30.0^\circ = 0.500$
9. Since the size of large eggs can vary by 10%, the random large egg used in a recipe has a size with an uncertainty of about  $\pm 5\%$ . Thus the amount of the other ingredients can also vary by about  $\pm 5\%$  and not adversely affect the recipe.
10. In estimating the number of car mechanics, the assumptions and estimates needed are:
  - the population of the city
  - the number of cars per person in the city
  - the number of cars that a mechanic can repair in a day
  - the number of days that a mechanic works in a year
  - the number of times that a car is taken to a mechanic, per year

We estimate that there is 1 car for every 2 people, that a mechanic can repair 3 cars per day, that a mechanic works 250 days a year, and that a car needs to be repaired twice per year.

- (a) For San Francisco, we estimate the population at one million people. The number of mechanics is found by the following calculation.

$$(1 \times 10^6 \text{ people}) \left( \frac{1 \text{ car}}{2 \text{ people}} \right) \left( \frac{2 \frac{\text{repairs}}{\text{year}}}{1 \text{ car}} \right) \left( \frac{1 \text{ yr}}{250 \text{ workdays}} \right) \left( \frac{1 \text{ mechanic}}{3 \frac{\text{repairs}}{\text{workday}}} \right) = \boxed{1300 \text{ mechanics}}$$

- (b) For Upland, Indiana, the population is about 4000. The number of mechanics is found by a similar calculation, and would be  $\boxed{5 \text{ mechanics}}$ . There are actually two repair shops in Upland, employing a total of 6 mechanics.

## Solutions to Problems

1. (a) 14 billion years =  $\boxed{1.4 \times 10^{10} \text{ years}}$   
 (b)  $(1.4 \times 10^{10} \text{ y})(3.156 \times 10^7 \text{ s/y}) = \boxed{4.4 \times 10^{17} \text{ s}}$
2. (a) 214  $\boxed{3 \text{ significant figures}}$   
 (b) 81.60  $\boxed{4 \text{ significant figures}}$   
 (c) 7.03  $\boxed{3 \text{ significant figures}}$   
 (d) 0.03  $\boxed{1 \text{ significant figure}}$   
 (e) 0.0086  $\boxed{2 \text{ significant figures}}$   
 (f) 3236  $\boxed{4 \text{ significant figures}}$   
 (g) 8700  $\boxed{2 \text{ significant figures}}$

3. (a)  $1.156 = \boxed{1.156 \times 10^0}$

(b)  $21.8 = \boxed{2.18 \times 10^1}$

(c)  $0.0068 = \boxed{6.8 \times 10^{-3}}$

(d)  $27.635 = \boxed{2.7635 \times 10^1}$

(e)  $0.219 = \boxed{2.19 \times 10^{-1}}$

(f)  $444 = \boxed{4.44 \times 10^2}$

4. (a)  $8.69 \times 10^4 = \boxed{86,900}$

(b)  $9.1 \times 10^3 = \boxed{9,100}$

(c)  $8.8 \times 10^{-1} = \boxed{0.88}$

(d)  $4.76 \times 10^2 = \boxed{476}$

(e)  $3.62 \times 10^{-5} = \boxed{0.0000362}$

5. The uncertainty is taken to be 0.01 m.

$$\% \text{ uncertainty} = \frac{0.01 \text{ m}}{1.57 \text{ m}} \times 100\% = \boxed{1\%}$$

6.  $\% \text{ uncertainty} = \frac{0.25 \text{ m}}{3.76 \text{ m}} \times 100\% = \boxed{6.6\%}$

7. (a)  $\% \text{ uncertainty} = \frac{0.2 \text{ s}}{5 \text{ s}} \times 100\% = \boxed{4\%}$

(b)  $\% \text{ uncertainty} = \frac{0.2 \text{ s}}{50 \text{ s}} \times 100\% = \boxed{0.4\%}$

(c)  $\% \text{ uncertainty} = \frac{0.2 \text{ s}}{300 \text{ s}} \times 100\% = \boxed{0.07\%}$

8. To add values with significant figures, adjust all values to be added so that their exponents are all the same.

$$9.2 \times 10^3 \text{ s} + 8.3 \times 10^4 \text{ s} + 0.008 \times 10^6 \text{ s} = 9.2 \times 10^3 \text{ s} + 83 \times 10^3 \text{ s} + 8 \times 10^3 \text{ s} = (9.2 + 83 + 8) \times 10^3 \text{ s}$$

$$= 100 \times 10^3 \text{ s} = \boxed{1.00 \times 10^5 \text{ s}}$$

When adding, keep the least accurate value, and so keep to the “ones” place in the parentheses.

9.  $(2.079 \times 10^2 \text{ m})(0.082 \times 10^{-1}) = \boxed{1.7 \text{ m}}$ . When multiplying, the result should have as many digits as the number with the least number of significant digits used in the calculation.

10. To find the approximate uncertainty in the area, calculate the area for the specified radius, the minimum radius, and the maximum radius. Subtract the extreme areas. The uncertainty in the area is then half this variation in area. The uncertainty in the radius is assumed to be  $0.1 \times 10^4 \text{ cm}$ .

$$A_{\text{specified}} = \pi r_{\text{specified}}^2 = \pi (3.8 \times 10^4 \text{ cm})^2 = 4.5 \times 10^9 \text{ cm}^2$$

$$A_{\text{min}} = \pi r_{\text{min}}^2 = \pi (3.7 \times 10^4 \text{ cm})^2 = 4.30 \times 10^9 \text{ cm}^2$$

$$A_{\text{max}} = \pi r_{\text{max}}^2 = \pi (3.9 \times 10^4 \text{ cm})^2 = 4.78 \times 10^9 \text{ cm}^2$$

$$\Delta A = \frac{1}{2}(A_{\text{max}} - A_{\text{min}}) = \frac{1}{2}(4.78 \times 10^9 \text{ cm}^2 - 4.30 \times 10^9 \text{ cm}^2) = 0.24 \times 10^9 \text{ cm}^2$$

Thus the area should be quoted as  $A = (4.5 \pm 0.2) \times 10^9 \text{ cm}^2$

11. To find the approximate uncertainty in the volume, calculate the volume for the specified radius, the minimum radius, and the maximum radius. Subtract the extreme volumes. The uncertainty in the volume is then half this variation in volume.

$$V_{\text{specified}} = \frac{4}{3}\pi r_{\text{specified}}^3 = \frac{4}{3}\pi (2.86 \text{ m})^3 = 9.80 \times 10^1 \text{ m}^3$$

$$V_{\text{min}} = \frac{4}{3}\pi r_{\text{min}}^3 = \frac{4}{3}\pi (2.77 \text{ m})^3 = 8.903 \times 10^1 \text{ m}^3$$

$$V_{\text{max}} = \frac{4}{3}\pi r_{\text{max}}^3 = \frac{4}{3}\pi (2.95 \text{ m})^3 = 10.754 \times 10^1 \text{ m}^3$$

$$\Delta V = \frac{1}{2}(V_{\text{max}} - V_{\text{min}}) = \frac{1}{2}(10.754 \times 10^1 \text{ m}^3 - 8.903 \times 10^1 \text{ m}^3) = 0.926 \times 10^1 \text{ m}^3$$

The percent uncertainty is  $\frac{\Delta V}{V_{\text{specified}}} = \frac{0.923 \times 10^1 \text{ m}^3}{9.80 \times 10^1 \text{ m}^3} \times 100 = 0.09444 = 9\%$

- |         |                  |                                  |  |
|---------|------------------|----------------------------------|--|
| 12. (a) | 286.6 mm         | $286.6 \times 10^{-3} \text{ m}$ | $0.286 \text{ 6 m}$                                  |
| (b)     | $85 \mu\text{V}$ | $85 \times 10^{-6} \text{ V}$    | $0.000 \text{ 085 V}$                                |
| (c)     | 760 mg           | $760 \times 10^{-6} \text{ kg}$  | $0.000 \text{ 760 kg}$ (if last zero is significant) |
| (d)     | 60.0 ps          | $60.0 \times 10^{-12} \text{ s}$ | $0.000 \text{ 000 000 0600 s}$                       |
| (e)     | 22.5 fm          | $22.5 \times 10^{-15} \text{ m}$ | $0.000 \text{ 000 000 000 022 5 m}$                  |
| (f)     | 2.50 gigavolts   | $2.5 \times 10^9 \text{ volts}$  | $2,500,000,000 \text{ volts}$                        |

- |         |                                   |   |
|---------|-----------------------------------|---|
| 13. (a) | $1 \times 10^6 \text{ volts}$     | $1 \text{ megavolt} = 1 \text{ Mvolt}$      |
| (b)     | $2 \times 10^{-6} \text{ meters}$ | $2 \text{ micrometers} = 2 \mu\text{m}$     |
| (c)     | $6 \times 10^3 \text{ days}$      | $6 \text{ kilodays} = 6 \text{ kdays}$      |
| (d)     | $18 \times 10^2 \text{ bucks}$    | $18 \text{ hectobucks} = 18 \text{ hbucks}$ |
| (e)     | $8 \times 10^{-9} \text{ pieces}$ | $8 \text{ nanopieces} = 8 \text{ npieces}$  |

14. (a) Assuming a height of 5 feet 10 inches, then  $5'10" = (70 \text{ in})(1 \text{ m}/39.37 \text{ in}) = 1.8 \text{ m}$
- (b) Assuming a weight of 165 lbs, then  $(165 \text{ lbs})(0.456 \text{ kg}/1 \text{ lb}) = 75.2 \text{ kg}$

Technically, pounds and mass measure two separate properties. To make this conversion, we have to assume that we are at a location where the acceleration due to gravity is  $9.8 \text{ m/s}^2$ .

15. (a)  $93 \text{ million miles} = (93 \times 10^6 \text{ miles})(1610 \text{ m}/1 \text{ mile}) = \boxed{1.5 \times 10^{11} \text{ m}}$

(b)  $1.5 \times 10^{11} \text{ m} = 150 \times 10^9 \text{ m} = \boxed{150 \text{ gigameters}}$  or  $1.5 \times 10^{11} \text{ m} = 0.15 \times 10^{12} \text{ m} = \boxed{0.15 \text{ terameters}}$

16. (a)  $1 \text{ ft}^2 = (1 \text{ ft}^2)(1 \text{ yd}/3 \text{ ft})^2 = \boxed{0.111 \text{ yd}^2}$

(b)  $1 \text{ m}^2 = (1 \text{ m}^2)(3.28 \text{ ft}/1 \text{ m})^2 = \boxed{10.8 \text{ ft}^2}$

17. Use the speed of the airplane to convert the travel distance into a time.

$$1.00 \text{ km} \left( \frac{1 \text{ h}}{950 \text{ km}} \right) \left( \frac{3600 \text{ s}}{1 \text{ h}} \right) = \boxed{3.8 \text{ s}}$$

18. (a)  $1.0 \times 10^{-10} \text{ m} = (1.0 \times 10^{-10} \text{ m})(39.37 \text{ in}/1 \text{ m}) = \boxed{3.9 \times 10^{-9} \text{ in}}$

(b)  $(1.0 \text{ cm}) \left( \frac{1 \text{ m}}{100 \text{ cm}} \right) \left( \frac{1 \text{ atom}}{1.0 \times 10^{-10} \text{ m}} \right) = \boxed{1.0 \times 10^8 \text{ atoms}}$

19. To add values with significant figures, adjust all values to be added so that their units are all the same.

$$1.80 \text{ m} + 142.5 \text{ cm} + 5.34 \times 10^5 \mu\text{m} = 1.80 \text{ m} + 1.425 \text{ m} + 0.534 \text{ m} = 3.759 \text{ m} = \boxed{3.76 \text{ m}}$$

When adding, the final result is to be no more accurate than the least accurate number used. In this case, that is the first measurement, which is accurate to the hundredths place.

20. (a)  $(1 \text{ k}/\text{h}) \left( \frac{0.621 \text{ mi}}{1 \text{ km}} \right) = \boxed{0.621 \text{ mi}/\text{h}}$

(b)  $(1 \text{ m}/\text{s}) \left( \frac{3.28 \text{ ft}}{1 \text{ m}} \right) = \boxed{3.28 \text{ ft}/\text{s}}$

(c)  $(1 \text{ km}/\text{h}) \left( \frac{1000 \text{ m}}{1 \text{ km}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) = \boxed{0.278 \text{ m}/\text{s}}$

21. One mile is  $1.61 \times 10^3 \text{ m}$ . It is 110 m longer than a 1500-m race. The percentage difference is

$$\frac{110 \text{ m}}{1500 \text{ m}} \times 100\% = \boxed{7.3\%}$$

22. (a)  $1.00 \text{ ly} = (2.998 \times 10^8 \text{ m}/\text{s})(3.156 \times 10^7 \text{ s}) = \boxed{9.46 \times 10^{15} \text{ m}}$

(b)  $(1.00 \text{ ly}) \left( \frac{9.462 \times 10^{15} \text{ m}}{1.00 \text{ ly}} \right) \left( \frac{1 \text{ AU}}{1.50 \times 10^{11} \text{ m}} \right) = \boxed{6.31 \times 10^4 \text{ AU}}$

(c)  $(2.998 \times 10^8 \text{ m}/\text{s}) \left( \frac{1 \text{ AU}}{1.50 \times 10^{11} \text{ m}} \right) \left( \frac{3600 \text{ s}}{1 \text{ hr}} \right) = \boxed{7.20 \text{ AU}/\text{h}}$

23. The surface area of a sphere is found by  $A = 4\pi r^2 = 4\pi (d/2)^2 = \pi d^2$ .

$$(a) \quad A_{\text{Moon}} = \pi D_{\text{Moon}}^2 = \pi (3.48 \times 10^6 \text{ m})^2 = \boxed{3.80 \times 10^{13} \text{ m}^2}$$

$$(b) \quad \frac{A_{\text{Earth}}}{A_{\text{Moon}}} = \frac{\pi D_{\text{Earth}}^2}{\pi D_{\text{Moon}}^2} = \left( \frac{D_{\text{Earth}}}{D_{\text{Moon}}} \right)^2 = \left( \frac{R_{\text{Earth}}}{R_{\text{Moon}}} \right)^2 = \left( \frac{6.38 \times 10^6 \text{ m}}{1.74 \times 10^6 \text{ m}} \right)^2 = \boxed{13.4}$$

$$24. (a) \quad 2800 = 2.8 \times 10^3 \approx 1 \times 10^3 = \boxed{10^3}$$

$$(b) \quad 86.30 \times 10^2 = 8.630 \times 10^3 \approx 10 \times 10^3 = \boxed{10^4}$$

$$(c) \quad 0.0076 = 7.6 \times 10^{-3} \approx 10 \times 10^{-3} = \boxed{10^{-2}}$$

$$(d) \quad 15.0 \times 10^8 = 1.5 \times 10^9 \approx 1 \times 10^9 = \boxed{10^9}$$

25. The textbook is approximately 20 cm deep and 4 cm wide. With books on both sides of a shelf, with a little extra space, the shelf would need to be about 50 cm deep. If the aisle is 1.5 meter wide, then about 1/4 of the floor space is covered by shelving. The number of books on a single shelf level is then  $\frac{1}{4}(3500 \text{ m}^2) \left( \frac{1 \text{ book}}{(0.25 \text{ m})(0.04 \text{ m})} \right) = 8.75 \times 10^4 \text{ books}$ . With 8 shelves of books, the total number of books stored is as follows.

$$\left( 8.75 \times 10^4 \frac{\text{books}}{\text{shelf level}} \right) (8 \text{ shelves}) \approx \boxed{7 \times 10^5 \text{ books}}.$$

26. The distance across the United States is about 3000 miles.

$$(3000 \text{ mi})(1 \text{ km}/0.621 \text{ mi})(1 \text{ hr}/10 \text{ km}) \approx \boxed{500 \text{ hr}}$$

Of course, it would take more time on the clock for the runner to run across the U.S. The runner could obviously not run for 500 hours non-stop. If they could run for 5 hours a day, then it would take about 100 days for them to cross the country.

27. An NCAA-regulation football field is 360 feet long (including the end zones) and 160 feet wide, which is about 110 meters by 50 meters, or  $5500 \text{ m}^2$ . The mower has a cutting width of 0.5 meters. Thus the distance to be walked is

$$d = \frac{\text{Area}}{\text{width}} = \frac{5500 \text{ m}^2}{0.5 \text{ m}} = 11000 \text{ m} = 11 \text{ km}$$

At a speed of 1 km/hr, then it will take about  $\boxed{11 \text{ h}}$  to mow the field.

28. A commonly accepted measure is that a person should drink eight 8-oz. glasses of water each day. That is about 2 quarts, or 2 liters of water per day. Then approximate the lifetime as 70 years.

$$(70 \text{ y})(365 \text{ d}/1 \text{ y})(2 \text{ L}/1 \text{ d}) \approx \boxed{5 \times 10^4 \text{ L}}$$

29. Consider the body to be a cylinder, about 170 cm tall, and about 12 cm in cross-sectional radius (a 30-inch waist). The volume of a cylinder is given by the area of the cross section times the height.

$$V = \pi r^2 h = \pi (12 \text{ cm})^2 (170 \text{ cm}) = 9 \times 10^4 \text{ cm}^3 \approx \boxed{8 \times 10^4 \text{ cm}^3}$$

30. Estimate one side of a house to be about 40 feet long, and about 10 feet high. Then the wall area of that particular wall is  $400 \text{ ft}^2$ . There would perhaps be 4 windows in that wall, each about 3 ft wide and 4 feet tall, so  $12 \text{ ft}^2$  per window, or about  $50 \text{ ft}^2$  of window per wall. Thus the percentage of wall area that is window area is  $\frac{50 \text{ ft}^2}{400 \text{ ft}^2} \times 100 = 12.5\%$ . Thus a rough estimate would be **10%–15%** of the house's outside wall area.

31. Assume that the tires last for 5 years, and so there is a tread wearing of  $0.2 \text{ cm/year}$ . Assume the average tire has a radius of  $40 \text{ cm}$ , and a width of  $10 \text{ cm}$ . Thus the volume of rubber that is becoming pollution each year from one tire is the surface area of the tire, times the thickness per year that is wearing. Also assume that there are  $150,000,000$  automobiles in the country – approximately one automobile for every two people. So the mass wear per year is given by

$$\begin{aligned} \left( \frac{\text{Mass}}{\text{year}} \right) &= \left( \frac{\text{Surface area}}{\text{tire}} \right) \left( \frac{\text{Thickness wear}}{\text{year}} \right) (\text{density of rubber}) (\# \text{ of tires}) \\ &= [2\pi(0.4 \text{ m})(0.1 \text{ m})] (0.002 \text{ m/y}) (1200 \text{ kg/m}^3) (600,000,000 \text{ tires}) \\ &= \boxed{4 \times 10^8 \text{ kg/y}} \end{aligned}$$

32. For the equation  $v = At^3 - Bt$ , the units of  $At^3$  must be the same as the units of  $v$ . So the units of  $A$  must be the same as the units of  $v/t^3$ , which would be **distance/time<sup>4</sup>**. Also, the units of  $Bt$  must be the same as the units of  $v$ . So the units of  $B$  must be the same as the units of  $v/t$ , which would be **distance/time<sup>2</sup>**.

33. (a) The quantity  $vt^2$  has units of  $(\text{m/s})(\text{s}^2) = \text{m} \cdot \text{s}$ , which do not match with the units of meters for  $x$ . The quantity  $2at$  has units of  $(\text{m/s}^2)(\text{s}) = \text{m/s}$ , which also do not match with the units of meters for  $x$ . Thus this equation **cannot be correct**.
- (b) The quantity  $v_0t$  has units of  $(\text{m/s})(\text{s}) = \text{m}$ , and  $\frac{1}{2}at^2$  has units of  $(\text{m/s}^2)(\text{s}^2) = \text{m}$ . Thus, since each term has units of meters, this equation **can be correct**.
- (c) The quantity  $v_0t$  has units of  $(\text{m/s})(\text{s}) = \text{m}$ , and  $2at^2$  has units of  $(\text{m/s}^2)(\text{s}^2) = \text{m}$ . Thus, since each term has units of meters, this equation **can be correct**.

34. The percentage accuracy is  $\frac{2 \text{ m}}{2 \times 10^7 \text{ m}} \times 100\% = \boxed{1 \times 10^{-5}\%}$ . The distance of  $20,000,000 \text{ m}$  needs to be distinguishable from  $20,000,002 \text{ m}$ , which means that **8 significant figures** are needed in the distance measurements.

35. Multiply the number of chips per wafer times the number of wafers that can be made from a cylinder.

$$\left( 100 \frac{\text{chips}}{\text{wafer}} \right) \left( \frac{1 \text{ wafer}}{0.60 \text{ mm}} \right) \left( \frac{300 \text{ mm}}{1 \text{ cylinder}} \right) = \boxed{50,000 \frac{\text{chips}}{\text{cylinder}}}$$

36. (a) # of seconds in 1.00 y:  $1.00 \text{ y} = (1.00 \text{ y}) \left( \frac{3.156 \times 10^7 \text{ s}}{1 \text{ y}} \right) = \boxed{3.16 \times 10^7 \text{ s}}$
- (b) # of nanoseconds in 1.00 y:  $1.00 \text{ y} = (1.00 \text{ y}) \left( \frac{3.156 \times 10^7 \text{ s}}{1 \text{ y}} \right) \left( \frac{1 \times 10^9 \text{ ns}}{1 \text{ s}} \right) = \boxed{3.16 \times 10^{16} \text{ ns}}$
- (c) # of years in 1.00 s:  $1.00 \text{ s} = (1.00 \text{ s}) \left( \frac{1 \text{ y}}{3.156 \times 10^7 \text{ s}} \right) = \boxed{3.17 \times 10^{-8} \text{ y}}$

37. Assume that the alveoli are spherical, and that the volume of a typical human lung is about 2 liters, which is  $.002 \text{ m}^3$ . The diameter can be found from the volume of a sphere,  $\frac{4}{3}\pi r^3$ .

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi (d/2)^3 = \frac{\pi d^3}{6}$$

$$(3 \times 10^8) \pi \frac{d^3}{6} = 2 \times 10^{-3} \text{ m}^3 \rightarrow d = \left[ \frac{6(2 \times 10^{-3})}{3 \times 10^8 \pi} \text{ m}^3 \right]^{1/3} = \boxed{2 \times 10^{-4} \text{ m}}$$

38. 1 hectare = (1 hectare)  $\left( \frac{10^4 \text{ m}^2}{1 \text{ hectare}} \right) \left( \frac{3.28 \text{ ft}}{1 \text{ m}} \right)^2 \left( \frac{1 \text{ acre}}{4 \times 10^4 \text{ ft}^2} \right) = \boxed{2.69 \text{ acres}}$

39. (a)  $\left( \frac{10^{-15} \text{ kg}}{1 \text{ bacterium}} \right) \left( \frac{1 \text{ proton or neutron}}{10^{-27} \text{ kg}} \right) = \boxed{10^{12} \text{ protons or neutrons}}$
- (b)  $\left( \frac{10^{-17} \text{ kg}}{1 \text{ DNA molecule}} \right) \left( \frac{1 \text{ proton or neutron}}{10^{-27} \text{ kg}} \right) = \boxed{10^{10} \text{ protons or neutrons}}$
- (c)  $\left( \frac{10^2 \text{ kg}}{1 \text{ human}} \right) \left( \frac{1 \text{ proton or neutron}}{10^{-27} \text{ kg}} \right) = \boxed{10^{29} \text{ protons or neutrons}}$
- (d)  $\left( \frac{10^{41} \text{ kg}}{1 \text{ galaxy}} \right) \left( \frac{1 \text{ proton or neutron}}{10^{-27} \text{ kg}} \right) = \boxed{10^{68} \text{ protons or neutrons}}$

40. There are about 300,000,000 people in the United States. Assume that half of them have cars, that they each drive 12,000 miles per year, and their cars get 20 miles per gallon of gasoline.

$$(3 \times 10^8 \text{ people}) \left( \frac{1 \text{ automobile}}{2 \text{ people}} \right) \left( \frac{12,000 \text{ mi}}{1 \text{ y}} \right) \left( \frac{1 \text{ gallon}}{20 \text{ mi}} \right) \approx \boxed{1 \times 10^{11} \text{ gallons/y}}$$

41. Approximate the gumball machine as a rectangular box with a square cross-sectional area. In counting gumballs across the bottom, there are about 10 in a row. Thus we estimate that one layer contains about 100 gumballs. In counting vertically, we see that there are about 15 rows. Thus we estimate that there are about  $\boxed{1500 \text{ gumballs}}$  in the machine.

42. The volume of water used by the people can be calculated as follows:

$$(4 \times 10^4 \text{ people}) \left( \frac{1200 \text{ L/day}}{4 \text{ people}} \right) \left( \frac{365 \text{ day}}{1 \text{ y}} \right) \left( \frac{1000 \text{ cm}^3}{1 \text{ L}} \right) \left( \frac{1 \text{ km}}{10^5 \text{ cm}} \right)^3 = 4.4 \times 10^{-3} \text{ km}^3/\text{y}$$



The depth of water is found by dividing the volume by the area.

$$d = \frac{V}{A} = \frac{4.4 \times 10^{-3} \text{ km}^3/\text{y}}{50 \text{ km}^2} = \left( 8.76 \times 10^{-5} \frac{\text{km}}{\text{y}} \right) \left( \frac{10^5 \text{ cm}}{1 \text{ km}} \right) = 8.76 \text{ cm/y} \approx \boxed{9 \text{ cm/y}}$$

43. The volume of a sphere is given by  $V = \frac{4}{3}\pi r^3$ . For our 1-ton rock, we can calculate the volume to be

$$V = (1 \text{ T}) \left( \frac{2000 \text{ lb}}{1 \text{ T}} \right) \left( \frac{1 \text{ ft}^3}{186 \text{ lb}} \right) = 10.8 \text{ ft}^3.$$

Then the radius is found by

$$d = 2r = 2 \left( \frac{3V}{4\pi} \right)^{1/3} = 2 \left[ \frac{3(10.8 \text{ ft}^3)}{4\pi} \right]^{1/3} = 2.74 \text{ ft} \approx \boxed{3 \text{ ft}}$$

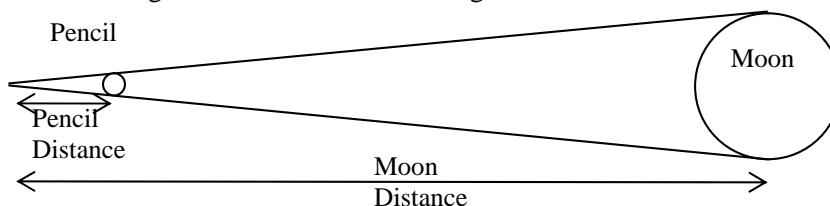
44. To calculate the mass of water, we need to find the volume of water, and then convert the volume to mass.

$$\left[ (4 \times 10^1 \text{ km}^2) \left( \frac{10^5 \text{ cm}}{1 \text{ km}} \right)^2 \right] (1.0 \text{ cm}) \left( \frac{10^{-3} \text{ kg}}{1 \text{ cm}^3} \right) \left( \frac{1 \text{ ton}}{10^3 \text{ kg}} \right) = \boxed{4 \times 10^5 \text{ ton}}$$

To find the number of gallons, convert the volume to gallons.

$$\left[ (4 \times 10^1 \text{ km}^2) \left( \frac{10^5 \text{ cm}}{1 \text{ km}} \right)^2 \right] (1.0 \text{ cm}) \left( \frac{1 \text{ L}}{1 \times 10^3 \text{ cm}^3} \right) \left( \frac{1 \text{ gal}}{3.78 \text{ L}} \right) = \boxed{1 \times 10^8 \text{ gal}}$$

45. A pencil has a diameter of about 0.7 cm. If held about 0.75 m from the eye, it can just block out the Moon. The ratio of pencil diameter to arm length is the same as the ratio of Moon diameter to Moon distance. From the diagram, we have the following ratios.



$$\frac{\text{Pencil diameter}}{\text{Pencil distance}} = \frac{\text{Moon diameter}}{\text{Moon distance}} \rightarrow$$

$$\text{Moon diameter} = \frac{\text{pencil diameter}}{\text{pencil distance}} (\text{Moon distance}) = \frac{7 \times 10^{-3} \text{ m}}{0.75 \text{ m}} (3.8 \times 10^5 \text{ km}) \approx \boxed{3500 \text{ km}}$$

46. The person walks 4 km/h, 10 hours each day. The radius of the Earth is about 6380 km, and the distance around the world at the equator is the circumference,  $2\pi R_{\text{Earth}}$ . We assume that the person can “walk on water”, and so ignore the existence of the oceans.

$$2\pi (6380 \text{ km}) \left( \frac{1 \text{ h}}{4 \text{ km}} \right) \left( \frac{1 \text{ d}}{10 \text{ h}} \right) = \boxed{1 \times 10^3 \text{ d}}$$

47. A cubit is about a half of a meter, by measuring several people's forearms. Thus the dimensions of Noah's ark would be  $\boxed{150 \text{ m long, } 25 \text{ m wide, } 15 \text{ m high}}$ . The volume of the ark is found by multiplying the three dimensions.

$$V = (150 \text{ m})(25 \text{ m})(15 \text{ m}) = 5.625 \times 10^4 \text{ m}^3 \approx \boxed{6 \times 10^4 \text{ m}^3}$$

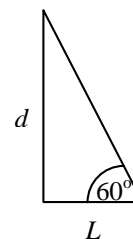
48. The volume of the oil will be the area times the thickness. The area is  $\pi r^2 = \pi (d/2)^2$ , and so

$$V = \pi (d/2)^2 t \rightarrow d = 2\sqrt{\frac{V}{\pi t}} = 2\sqrt{\frac{1000 \text{ cm}^3 \left(\frac{1 \text{ m}}{100 \text{ cm}}\right)^3}{\pi (2 \times 10^{-10} \text{ m})}} = \boxed{3 \times 10^3 \text{ m}}.$$

49. Consider the diagram shown.  $L$  is the distance she walks upstream, which is about 120 yards. Find the distance across the river from the diagram.

$$\tan 60^\circ = \frac{d}{L} \rightarrow d = L \tan 60^\circ = (120 \text{ yd}) \tan 60^\circ = \boxed{210 \text{ yd}}$$

$$(210 \text{ yd}) \left(\frac{3 \text{ ft}}{1 \text{ yd}}\right) \left(\frac{0.305 \text{ m}}{1 \text{ ft}}\right) = \boxed{190 \text{ m}}$$



$$50. \left(\frac{8 \text{ s}}{1 \text{ y}}\right) \left(\frac{1 \text{ y}}{3.156 \times 10^7 \text{ s}}\right) \times 100\% = \boxed{3 \times 10^{-5} \%}$$

- 51.** The volume of a sphere is found by  $V = \frac{4}{3} \pi r^3$ .

$$V_{\text{Moon}} = \frac{4}{3} \pi R_{\text{Moon}}^3 = \frac{4}{3} \pi (1.74 \times 10^6 \text{ m})^3 = \boxed{2.21 \times 10^{19} \text{ m}^3}$$

$$\frac{V_{\text{Earth}}}{V_{\text{Moon}}} = \frac{\frac{4}{3} \pi R_{\text{Earth}}^3}{\frac{4}{3} \pi R_{\text{Moon}}^3} = \left(\frac{R_{\text{Earth}}}{R_{\text{Moon}}}\right)^3 = \left(\frac{6.38 \times 10^6 \text{ m}}{1.74 \times 10^6 \text{ m}}\right)^3 = 49.3.$$

Thus it would take about  $\boxed{49.3}$  Moons to create a volume equal to that of the Earth.

$$52. (a) 1.0 \text{ \AA} = (1.0 \text{ \AA}) \left(\frac{10^{-10} \text{ m}}{1 \text{ \AA}}\right) \left(\frac{1 \text{ nm}}{10^{-9} \text{ m}}\right) = \boxed{0.10 \text{ nm}}$$

$$(b) 1.0 \text{ \AA} = (1.0 \text{ \AA}) \left(\frac{10^{-10} \text{ m}}{1 \text{ \AA}}\right) \left(\frac{1 \text{ fm}}{10^{-15} \text{ m}}\right) = \boxed{1.0 \times 10^5 \text{ fm}}$$

$$(c) 1.0 \text{ m} = (1.0 \text{ m}) \left(\frac{1 \text{ \AA}}{10^{-10} \text{ m}}\right) = \boxed{1.0 \times 10^{10} \text{ \AA}}$$

$$(d) 1.0 \text{ ly} = (1.0 \text{ ly}) \left(\frac{9.46 \times 10^{15} \text{ m}}{1 \text{ ly}}\right) \left(\frac{1 \text{ \AA}}{10^{-10} \text{ m}}\right) = \boxed{9.5 \times 10^{25} \text{ \AA}}$$

53. (a) Note that  $\sin 15.0^\circ = 0.259$  and  $\sin 15.5^\circ = 0.267$ .

$$\left(\frac{\Delta\theta}{\theta}\right)100 = \left(\frac{0.5^\circ}{15.0^\circ}\right)100 = \boxed{3\%} \qquad \left(\frac{\Delta \sin \theta}{\sin \theta}\right)100 = \left(\frac{8 \times 10^{-3}}{0.259}\right)100 = \boxed{3\%}$$

- (b) Note that  $\sin 75.0^\circ = 0.966$  and  $\sin 75.5^\circ = 0.968$ .

$$\left(\frac{\Delta\theta}{\theta}\right)100 = \left(\frac{0.5^\circ}{75.0^\circ}\right)100 = \boxed{0.7\%} \qquad \left(\frac{\Delta \sin \theta}{\sin \theta}\right)100 = \left(\frac{2 \times 10^{-3}}{0.966}\right)100 = \boxed{0.2\%}$$

A consequence of this result is that when using a protractor, and you have a fixed uncertainty in the angle ( $\pm 0.5^\circ$  in this case), you should measure the angles from a reference line that gives a large angle measurement rather than a small one. Note above that the angles around  $75^\circ$  had only a 0.2% error in  $\sin \theta$ , while the angles around  $15^\circ$  had a 3% error in  $\sin \theta$ .

54. Utilize the fact that walking totally around the Earth along the meridian would trace out a circle whose full  $360^\circ$  would equal the circumference of the Earth.

$$(1 \text{ minute}) \left( \frac{1^\circ}{60 \text{ minute}} \right) \left( \frac{2\pi(6.38 \times 10^3 \text{ km})}{360^\circ} \right) \left( \frac{0.621 \text{ m}}{1 \text{ km}} \right) = \boxed{1.15 \text{ mi}}$$